PISCATAWAY
TOWNSHIP SCHOOLS

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Honors Math 4

Content Area: Mathematics
Grade Span: 5
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COURSE OVERVIEW

Description

This course aims to supplement and enrich instruction for students exceeding expectations in grade level mathematics coursework.

Goals

Operations and Algebraic Thinking

- Write and interpret numerical expressions.
- Analyze patterns and relationships.

Number and Operations in Base Ten

- Understand the place value system.
- Perform operations with multi-digit whole numbers and with decimals to hundredths.

Number and Operations—Fractions

- Use equivalent fractions as a strategy to add and subtract fractions.
- Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Measurement and Data

- Convert like measurement units within a given measurement system.
- Represent and interpret data.
- Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

Geometry

- Graph points on the coordinate plane to solve real-world and mathematical problems.
- Classify two-dimensional figures into categories based on their properties.

Mathematical Practices

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

Scope and Sequence

Unit	Topic	Length
Unit 1	At The Mall with Algebra: Working with Variables and Equations	34 days
Unit 2	Getting Into Shapes: Exploring Relationships Among 2-D and 3-D Shapes	34 days

Resources

Core Text: Project M^3: Mentoring Mathematical Minds

Suggested Resources:

- www.Desmos.com
- https://www.flourishkh.com/
- www.schoology.com

UNIT 1: At the Mall with Algebra

Summary and Rationale

Although when we hear the word algebra, we generally think about the high school curriculum, the National Council of Teachers of Mathematics (2000) has elevated this content area to one of great importance across the K–12 mathematics curriculum. In fact, it is one of only five content standards. In the early grades, the emphasis should be on developing algebraic reasoning, and that is what this unit emphasizes.

In many respects, algebra is the generalization of arithmetic, and so we encourage students to approach the study of variables, expressions and equations using their number sense, logical reasoning and problem-solving strategies. We introduce students to these concepts through interesting problem-solving situations. They are intrigued to figure out the mathematics behind number tricks and to solve variable puzzles. In discovering the answers to these problems, students learn about different ways to represent and solve similar types of problems using variables, expressions and equations.

As students represent and analyze mathematical situations using algebraic symbols, they come to understand the basic notions of equality and equivalent expressions. They learn how variables are used to represent change in quantities and also to represent a specific unknown in an equation. The idea that the same variable represents the same quantity in a given equation or set of equations is a fundamental algebraic concept that students will use throughout their mathematical learning. In this unit students' understanding of these concepts comes out of informal problem-solving in which they use mathematics to make sense of the situations posed, just like real mathematicians. We hope that the experiences and discussions in the unit will provide a rich context for introducing students to algebraic thinking while strengthening their problem-solving and mathematical communication skills.

This unit aims to develop students' understanding of the idea of a variable as an unknown quantity using a letter or symbol. It also challenges students to think deeply about the concept of equality as it relates to solving equations. Finally, the unit asks student to model and solve contextualized problems that involve two variables using a variety of strategies such as guess and test and make an organized list. The CCSS Content Standards (retrieved from http://www.corestandards.org/Math/) addressed in the unit are from grades 5-7.

Recommended Pacing

Unit Test given as Pretest 1 day
Chapter 1: Thinking about Variables and Equations
Lesson 1: I'm Thinking of a Number 4 days
Lesson 2: Number Tricks 4 days
Lesson 3: Variable Puzzles 4 days
Lesson 4: Cover Up! 4 days
Check-up 1 1/2 day
Chapter 2: Equations That Go Together
Lesson 1: Pet Parade 4 days
Lesson 2: A Penny for Your Thoughts 4 days
Lesson 3: Seasonal Symbols 4 days
Lesson 4: At the Mall 4 days

Check-up 2 1/2 day
Unit Test 1 day
Total 34 days
Pacing is based on a 45-50 min math class period 4 days per week.

State Standards

Standard: Operations and Algebraic Thinking (Grade 5)	
3	Analyze patterns and relationships.
Standard: Expressions and Equations (Grade 6 & 7)	
A2	Write, read, and evaluate expressions in which letters stand for numbers.
A2A	Write expressions that record operations with numbers and with letters standing for numbers.
B5	Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
B6	Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
B7	Solve real-world and mathematical problems by writing and solving equations of the form x + p = q and px = q for cases in which p, q and x are all nonnegative rational numbers.
B3-4	Solve real-life and mathematical problems using numerical and algebraic expressions and equations.
B4	Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

Instructional Focus

Unit Enduring Understandings

- Variables are used in different ways.
- There are multiple approaches to problem solving.
- Mathematical discourse develops and deepens understanding.

Unit Essential Questions

- What ways can variables be used?
- How can I engage in mathematical discourse?
- How can creativity be applied to problem solving?

Objectives

Students will know:

- the meaning for the term variable (specifically, a quantity whose value changes or varies)
- the difference between expressions and equations
- number problems presented in words using variables, expressions, and equations.
- strategies such as working backwards and inverse operations or “undoing” to find value for variables.
- equations can written using variables
- the characteristics of sums of even and odd numbers

- solutions to equations
- the same variables have the same values in the same equation
- making true statements involves replacing variables with numbers
- a variety of strategies and make inferences to solve problems
- multiple strategies to solve equations and problems that involve two unknowns
- solving problems that involve two unknowns utilizes multiple pieces of information
- the value of making an organized list
- commutative property and properties of zero can be used to solve equations that go together

Students will be able to:

- demonstrate understanding of the meaning of the term variable
- differentiate between expressions and equations
- represent number problems presented in words using variables, expressions, and equations
- apply strategies such as working backwards and inverse operations or "undoing" to find values for variables.
- write equations using variables
- identify the characteristics of the sums of even and odd numbers
- solve equations
- demonstrate understanding that the same variables have the same variables in the same equation
- replace variables with numbers to make true statements
- make inferences and use a variety of strategies to solve problems
- solve equations using the cover-up method
- use multiple pieces of information to solve problems that involve two unknowns
- use guess and test method along with an organized list to solve problems with two unknowns
- recognize the value of making an organized list
- use organized list to solve equations that go together
- understand that same variables represent same values in each equation
- use the strategy of drawing a diagram to solve problems with two unknowns
- use substitution to solve equations that go together

use the commutative property and properties of zero when solving equations that go together

Resources

Suggested Resources:

-Utilize https://www.flourishkh.com/ for interactive activities and printable resources

-Utilize Think Beyond and Think Deeply Cards with early finishers and Hint Cards for students who need support

- Utilize www.schoology.com to post journal responses

Online Games:

https://www.mathplayground.com/algebraicreasoning.html

https://www.mathplayground.com/wangdoodles.html

http://www.aaamath.com/equ723-evaluate-1variable.html

- Utilize www.engageny.org for additional activities

UNIT 2: Getting Into Shapes:
Exploring Relationships Among 2-D and 3-D Shapes

Summary and Rationale

Students will explore two- and three-dimensional shapes with a focus on their properties, relationships among them and spatial visualization. The reasoning skills that they build upon in this unit help them to develop an understanding of more complex geometric concepts.

In Chapter 1 students learn new, more specialized vocabulary and learn how to describe properties of shapes with this terminology, allowing for greater clarity and precision in their explanations. They move from describing properties to comparing and contrasting properties of two- and three-dimensional shapes by classifying them into different groups based on their properties.

To better understand how students grow in their understanding of shapes — and thus understand the development of the activities in this unit — it helps to examine a model of geometric thinking created by Pierre van Hiele and Dina van Hiele-Geldof, contemporaries and colleagues of Jean Piaget. Their model represents a five-level hierarchy of ways of understanding spatial ideas (van Hiele, 1999). The levels describe how one thinks and what types of geometric ideas one thinks about rather than the amount of knowledge someone has. At the lowest of the levels, visualization, figures are judged by their appearance alone. For example, a student might claim, "I know it is a rectangle because it looks like a box." At this level, if a student saw a tilted square they would not identify it as a square because it does not look like one. At the next level, the descriptive level, students focus on the properties of shapes. They recognize that an equilateral triangle has three congruent angles, three congruent sides and symmetry. However, the properties are not necessarily in any order. Ordering occurs at the next level, the informal deduction level. At this level students can put properties in order and use them to formulate definitions. They are able to see the relationships among shapes and group them into classifications. For example, they would recognize that a square is also a rectangle and that both of these shapes can be classified as parallelograms.

The last two levels, deduction and rigor, are much more advanced. Students at the deduction level generally are studying high-school geometry. They are able to work with abstract statements about geometric properties and begin to appreciate the structure of a system complete with definitions, postulates and theorems. Students who study college-level geometry as a branch of mathematical science and focus on the axiomatic system itself and its relationship to other systems are at the level of rigor.

The focus in this unit is on moving students from their initial level of geometric thinking to the third level, informal deduction. Students will examine shapes, describe them using their definitions and properties, and then find relationships among shapes based on these properties. They will classify and reclassify shapes according to different properties. These kinds of experiences help students develop the more sophisticated reasoning skills used in informal deduction.

In addition to studying physical models, students develop and use mental images. Beginning at a very early age, students display the ability to use spatial logic through simple tasks, such as manipulating building blocks. Early student opportunities shape their ability to use spatial visualization as a problem-solving tool. Activities like building jigsaw puzzles and finding objects in hidden pictures provide explorations using spatial orientation, perceptual constancy and eye-hand coordination skills. There are many Internet sites where students can

participate in spatial reasoning and logic exercises. For example, the National Council of Teachers of Mathematics hosts the Illuminations site (www.illuminations.nctm.org) where students can manipulate two- and three-dimensional shapes. They determine the faces, edges and vertices of shapes and paint surfaces while rotating the shapes in space.

In Chapter 2 students learn how to plot points as ordered pairs in all four quadrants and mentally transform a shape by changing its position or orientation. They use the coordinate grid to connect location to transformations. These activities are aimed at developing students' sense of spatial visualization. This, in turn, will help students make better sense of the world around them.

This unit develops understanding of spatial visualization through the exploration of relationships among two-dimensional shapes and three-dimensional shapes. The CCSS Content Standards (retrieved from http://www.corestandards.org/Math/) addressed in the unit are mainly from the domain Geometry (spanning Grades 4, 5, 6, and 7). The lessons also incorporate some standards from the domain The Number System in Grade 6.

Recommended Pacing

Unit Test given as Pretest 1 day
Chapter 1: The Shape of Things
Lesson 1: You Either Have It…Or You Don't! 5 days
Lesson 2: Triple Play with Shapes 5 days
Lesson 3: 3-D Shapes Inside Out 5 days
Check-up 1 1/2 day
Chapter 2: Spatial Visualization: In the Mind's Eye
Lesson 1: Do You See What I See? 5 days
Lesson 2: On the Grid 5 days
Lesson 3: It is All in What You See! 5 days
Check-up 2 ½ day
Unit Test given as Posttest 1 day
Total 34 days
Pacing is based on a 60-minute math class period per day.

State Standards

Standard: Geometry	
4.G.A.1	Draw points, lines, line segments, rays, angles and perpendicular and parallel lines. Identify these in two-dimensional shapes.
4.G.A.2	Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines or the presence or absence of angles of a specified size.
5.G.A.1	Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (origin) arranged to coincide with the zero on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understanding that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis, and x-coordinate, y-axis and the

	y-coordinate).
5.G.A.2	Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.
5.B.3	Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category.
5.B.4	Classify two-dimensional figures in a hierarchy based on properties.
6.G.A.3	Draw polygons in the coordinate plane given coordinates for the vertices.
7.G.A.3	Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.
8.G.A.3	Describe the effect of translations, reflections, and rotations on two-dimensional figures using coordinates.
Standard: Measurement and Data	
4.MD.C.5.a	An angle is measured with reference to a circle with its center at the common endpoint o the rays, by considering the fraction of the circular arc between the points where the two ray intersect the circle. An angle that turns through 1/360 of a circle is all a "one-degree angle", and can be used to measure angles.
4.MD.C.5.b	An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.
Standard: The Number System	
6.NS.C.6.B	Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pair differ only by signs, the locations of the points are related by reflections across one or both axes.
6.NS.C.6.C	Find and position pairs of integers on a coordinate plane.
6.NS.C.8	Solve mathematical problems by graphing points in all four quadrants on the coordinate plane.

Instructional Focus

Unit Enduring Understandings

- Comparisons create characteristics.
- Perspective builds understanding.

Unit Essential Questions

- Can comparisons create understanding?
- What is the impact of movement?

Objectives

Students will know:

- characteristics of the various quadrilaterals;
- characteristics of three-dimensional shapes;
- cross sections from three-dimensional shapes;
- the effects of reflections, translations, and rotations on a shape;
- how to identify the coordinates of an ordered pair (all four quadrants);
- names of three-dimensional shapes.

Students will be able to:

- identify and define characteristics of different quadrilaterals;
- compare and contrast characteristics of two-dimensional shapes and their subcategories;
- define characteristics of three-dimensional shapes, including prisms, pyramids, cylinders, and cones;
- compare and contrast characteristics of three-dimensional shapes;
- identify the cross section that results from slicing three-dimensional shapes;
- identify transformations (reflections, translations, and rotations);
- transform shapes using reflections, translations, and rotations (including on a coordinate grid);
- plot ordered pairs on a coordinate plane (all four quadrants);
- identify the coordinates of a graphed point (all four quadrants);
- identify the names of three-dimensional shapes.

Resources

Suggested Resources/Activities:

- Discussion - Which one doesn't belong? http://wodb.ca/shapes.html
- Create a Venn Diagram comparing the attributes of quadrilaterals.
- Practice classifying quadrilaterals - https://www.ixl.com/math/grade-6/classify-quadrilaterals
- Practice graphing ordered pairs in all four quadrants and identify the coordinates of a graphed ordered pair.
- Practice identifying the rigid motion - http://www.sheppardsoftware.com/mathgames/geometry/shapeshoot/TranslateShapesShoot.htm
- Practice identifying the steps (rigid motions) needed to map a pre-image to an image - https://www.ixl.com/math/grade-5/reflection-rotation-and-translation
- Practice identifying the image after a rigid motion - https://www.ixl.com/math/grade-8/identify-reflections-rotations-and-translations