PISCATAWAY
TOWNSHIP SCHOOLS

Dr. Frank Ranelli
Superintendent of Schools

Dr. William Baskerville
Assistant Superintendent

Honors Math 5

Content Area: Mathematics
Grade Span: 5
Revised by: Gaye McGee and Lorie Rizzo
Presented by: Rebecca Dayton
Supervisor of Mathematics PreK-6
Approval date: August 5, 2019

Ralph Johnson
Shantell Cherry
Jeffrey Fields
Nitang Patel
Shelia Hobson
Sarah Rashid

Piscataway Township Schools
1515 Stelton Road
Piscataway, NJ 08854-1332
732 572-2289, ext. 2561
Fax 732 572-1540
www.piscatawayschools.org

COURSE OVERVIEW

Description

This course aims to supplement and enrich instruction for students exceeding expectations in grade level mathematics coursework.

Goals

Operations and Algebraic Thinking

- Write and interpret numerical expressions.
- Analyze patterns and relationships.

Number and Operations in Base Ten

- Understand the place value system.
- Perform operations with multi-digit whole numbers and with decimals to hundredths.

Number and Operations—Fractions

- Use equivalent fractions as a strategy to add and subtract fractions.
- Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Measurement and Data

- Convert like measurement units within a given measurement system.
- Represent and interpret data.
- Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

Geometry

- Graph points on the coordinate plane to solve real-world and mathematical problems.
- Classify two-dimensional figures into categories based on their properties.

Mathematical Practices

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

Scope and Sequence

Unit	Topic	Length
Unit 1	Fun at the Carnival: Exploring Proportional Reasoning	34 days
Unit 2	Record Makers and Breakers: Analyzing Graphs, Tables, and Equations	34 days

Resources

Core Text: Project M^3: Mentoring Mathematical Minds
Suggested Resources:

- www.Desmos.com
- https://www.flourishkh.com/
- www.schoology.com

UNIT 1: Fun at the Carnival

Summary and Rationale

Two topics in mathematics, geometry and proportional reasoning, are readily apparent in the world around us, and practical applications abound. In this unit, Fun at the Carnival: Using Proportional Reasoning, students are introduced to ratios, rates, proportional reasoning, similarity and congruence. Students explore ratio as a comparison of two quantities. They discover that if the ratio between the corresponding side lengths and angle measures of two figures is 1:1 (i.e., the measures are identical), then the geometric figures are congruent. They learn that for two figures to be similar, their corresponding side lengths must be in proportion and their corresponding angles must be congruent. Students also explore the relationship between congruence and similarity, learning that all congruent figures are similar, but not all similar figures are congruent.

After learning these basic concepts, students explore applications of similarity and congruence (mainly enlargements and reductions) in Chapter 1. All the lessons in the entire unit revolve around the Carnival and Stretch the Clown. Stretch's body enlarges and shrinks depending on which day you visit the fun house at the carnival. Students have fun exploring body ratios using Stretch and his cousin, Juggles the Clown. Students also take a look at Stretch's miniature car collection and learn how important the concept of similarity is in creating reductions.

In Chapter 2, students become set designers for the Carnival's Fun House. They use their creativity and newfound knowledge of ratios and geometry to design drawings for new rooms and mirrors for the Fun House. They investigate how two- and three-dimensional shapes change when they are scaled up or down. In particular, they examine changes in perimeter, area, and volume when dimensions are changed. Some startling mathematical discoveries are made!

These investigations into the properties of and relationships among similar shapes afford students many opportunities to develop and evaluate conjectures inductively and deductively. Our focus on verbal and written communication throughout the unit prepares students for creating and understanding more formal proofs as they continue their study of geometry in the upper grades.

This unit aims to develop students' understanding of proportional reasoning and geometry by supporting students in making sense of ratios and using proportions to solve a range of problems. Students connect numbers and geometry as they explore congruence and similarity and use the concepts of scale factor and create scale drawings. The CCSS Content Standards (retrieved from http://www.corestandards.org/Math/) addressed in the unit are from grades 6-7.

Recommended Pacing

Unit Test given as Pretest 1 day
Chapter 1: Exploring Proportional Reasoning and Similarity
Lesson 1: Comparing with Ratios 4 days
Lesson 2: Nesting Figures 4 days
Lesson 3: Exploring Similarity 4 days
Lesson 4: Scaling Up and Down 4 days
Check-up 1 1/2 day
Chapter 2: Using Proportional Reasoning and Similarity

Lesson 1: Scale Drawings and Perimeters 4 days
Lesson 2: Growing Polygons: Areas of Similar Figures 4 days
Lesson 3: Fantastical Elastical Boxes: Examining Volume 4 days
Check-up 2 1/2 day
Lesson 4: Let the Fun Begin! 4 days
Unit Test given as Posttest 1 day
Total 34 days
Pacing is based on a 45/50 minute math class period per day.

State Standards

Standard: Ratios and Proportional Reasoning (Gr.6 & 7)

CPI #	Cumulative Progress Indicator (CPI)
A1	Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.
A2	Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship.
A3	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
A3.D	Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.
A2	Recognize and represent proportional relationships between quantities.
A2.A	Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
A2.C	Represent proportional relationships by equations.

Standard: Geometry (Grade 7)

CPI #	Cumulative Progress Indicator (CPI)
A1	Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
B6	Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Instructional Focus

Unit Enduring Understandings

- Mathematics can be explained in the oral or written form
- One representation may sometimes be more helpful than another; and, used together, multiple representations give a fuller understanding of a problem
- Everyday objects have a variety of attributes, each of which can be measured in many ways
- What we measure affects how we measure it
- Mathematics is based on patterns, relationships, and a defined set of rules that interconnect and explain all mathematical concepts and natural phenomena

- Spatial sense and various geometric terms, concepts, and properties are used to model, identify, interpret, and describe relationships as they exist in the world; the terms, properties, and concepts chosen are based on the situation and what is to be communicated.
- Geometric properties can be used to construct geometric figures.
- Geometric relationships provide a means to solve real-life problems

Unit Essential Questions

- What is the best way to communicate mathematically?
- How can the collection, organization, interpretation, and display of data be used to answer questions?
- What is the best way to measure?
- How can measurements be used to solve problems?
- How can change be best represented mathematically?
- What is the best way to use geometry?
- How can spatial relationships be described by careful use of geometric language?
- How do geometric relationships help in solving problems?

Objectives

Students will know:

- the meaning of equivalent ratios and be able to generate equivalent ratios
- some figures with corresponding congruent angles are not similar
- figures that are congruent are also classified as similar figures
- mathematical similarity includes the relationship between corresponding sides
- ratios can be formed by comparing side lengths
- how the scale factor affects the size of a similar figure
- the relationship between congruent figures and similar figures
- the ratio of perimeters in similar figures is equal to the ratio of their corresponding sides
- if you scale each dimension of a polygon by a scale factor of k, the area of the scaled polygon will be k^2 times the area of the original polygon
- about square numbers and how to write square numbers using exponents
- a relationship exists between the volumes of two similar figures that is related to the scale factor

Students will be able to:

- use ratios to compare quantities using words and symbols
- measure and draw segments and represent relationships using equivalent ratios
- discover that corresponding angles in similar figures are congruent
- measure angles accurately
- .expand their definition of mathematical similarity to include the relationship between corresponding sides
- form ratios by comparing corresponding side lengths
- determine if two or more shapes are similar
- construct a definition for similar figures
- use scale factors to draw enlargements and reductions on grid paper
- enlarge geometric figures by using scale factors and measuring angles
- construct scale drawings of two-dimensional shapes
- explore changes in perimeter when creating scale drawings
- demonstrate mathematical reasoning by generalizing patterns, making conjectures, and explaining their logic
- construct similar polygons using multiple copies of the original polygon
- explore geometric and numerical patterns related to similar polygons

- create enlargements and reductions of three-dimensional figures using physical models
- calculate the volumes of rectangular prisms
- generalize patterns, make conjectures, and provide logical arguments about scaling
- write review questions that can be used in the “Let the Fun Begin!” game
- play the game as a way to review concepts and skills learned in the unit

Resources

Core Text: Project M3

Suggested Resources:

- https://www.flourishkh.com
- Think Beyond and Think Deeply Cards
- Hint Cards for support
- www.schoology.com journal responses

UNIT 2: Record Makers and Breakers: Analyzing Graphs, Tables, and Equations

Summary and Rationale

Many adults will tell you that they have not used algebra since they had to take a course by that name in high school. Mathematicians, however, will argue that we should use algebraic reasoning every day. To understand this difference, we need to understand what is meant by algebraic reasoning. Mathematicians often define mathematics as the study of patterns. If that is the case, then algebraic reasoning might be defined as recognizing, analyzing, developing, and justifying generalizations about those patterns. In algebra, symbols are often used to represent these generalizations.

This unit is devoted to the study of patterns of change and relationships among quantities, major algebraic concepts that form an essential foundation for more advanced mathematics including algebra and calculus. This focus on thinking about relationships rather than simply finding answers is critical to students' success in solving problems and making sense of mathematics. As noted in the recommended Mathematics Teaching Practices from the 2014 NCTM Principles to Action Executive Summary, teachers should "implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies." Students find that the world records that are used throughout the unit are engaging, and they often research additional records to deepen their understanding. Many are encouraged to attempt setting their own world records and learn to collect and analyze data as they do this.

Chapter 1 focuses on analyzing change in situations beginning with a qualitative perspective, giving students the opportunity to look globally at what is happening throughout the entire situation. Plotting points on a coordinate graph helps students visualize relationships among the quantities under consideration. To ensure that students focus on the overall patterns, the x- and y-axes are not labeled with numbers in the beginning activities. Rather, students make sense of the situation by investigating the relationships between variables. They learn to recognize dependent and independent variables and then analyze whether or not there is a relationship among the variables. This begins with an activity in which they analyze a graph displaying the height and the number of points scored by professional basketball players. They are often surprised to discover that there is no apparent relationship among the variables for the data shown. This is followed by lessons in which students analyze situations that have a constant rate of change and realize that the graphs of these situations illustrate the linear relationship. They identify situations with constant and varying rates of change as well as those with starting points at different values. This chapter ends with a look at ratios, proportional relationships and unit rates, again using situations, tables, and graphs. Students begin to use expressions and equations to describe the relationships that they discover.

In Chapter 2, students continue to explore a variety of world records as they use more complex quantitative data to create graphs and analyze the rate of change in more depth, including examining slope, the y-intercept, and points of intersection. In the lessons in Chapter 2, both the x- and y-axes are labeled with numbers. Students extend their understanding of ratios and proportions to finding unit rate and writing explicit (finding a general rule for the output when given the input) and recursive (finding the next term when the previous one is known) rules for the linear functions that are described. Once again, they begin with situations and use tables, graphs, and equations in their analyses.

In this unit, students learn about algebra as a set of concepts tied to the representation of relationships through words, equations, tables, and graphs. They also learn about algebra as a style of mathematical thinking for formalizing patterns of change. They extend their notion of variable from a letter in an equation that represents a number to a broader definition, that of a quantity that sometimes varies or changes. They will learn how to identify these quantities as variables, or things that vary, in different situations. They will use these dependent and independent variables to represent situations graphically, in tables, and with expressions and equations. Students also will analyze changes in the situations, including the rate of change, by looking at the relationships between the variables throughout the given situation. They will comprehend how to interpret the y-intercept in the context of a given situation and discuss how the steepness of the line, or slope, describes the rate of change. These experiences, discussions, and written reflections provide opportunities for advancing mathematical understanding and are an important prelude to the more formal study of algebra that students will encounter later in middle and high school.

This unit develops and reinforces understanding of algebraic and proportional reasoning and builds an understanding of the significance of slope and y-intercept. Students recognize dependent and independent variables, and they analyze rates of change to distinguish linear from non-linear patterns using situations, words, tables, graphs, and equations. The CCSS Content Standards (retrieved from http://www.corestandards.org/Math/) addressed in the unit come from the domains of Operations and Algebraic Thinking, Numbers and Operations in Base 10, and Geometry in fifth grade and from the sixth and seventh grade domains of Ratios and Proportional Reasoning and Expressions and Equations.

Recommended Pacing

Unit Test given as Pretest 1 day
Chapter 1: Change Artists
Lesson 1: Picture This! 4 days
Lesson 2: A Sweet-Smelling Walk 4 days
Lesson 3: Wacky World Records 4days
Lesson 4: It's a Dog's World 4 days
Check-up 1 1/2 days
Chapter 2: Records Are Made to Be Broken
Lesson 1: Walking on Water 4 days
Lesson 2: The Chain Gang 4 days
Lesson 3: A Penny Saved 4 days
Lesson 4: Set Your Own Record! 4 days
Check-up 2 1/2 days
Unit Test 1 day
Total 34 days
Pacing is based on a 45/50 minute math class period per day.

State Standards

Standard: Operations and Algebraic Thinking (Gr.5)

B.3	Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane.

Standard: Number and Operations in Base 10 (Gr.5)	
B.5	Fluently multiply multi-digit whole numbers using the standard algorithm.
B.6	Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
Standard: Geometry (Gr.5)	
A.1	Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).
A.2	Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane and interpret coordinate values of points in the context of the situation.
Standard: Ratios and Proportional Reasoning (Gr.6 & 7)	
A.1	Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.
A.2	Understand the concept of a unit rate a/b associated with a ratio a:b with b not equaling zero, and use rate language in the context of a ratio relationship.
A.3	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
A.1	Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.
A.2	Recognize and represent proportional relationships between quantities.
Standard: Expressions and Equations (Gr.6)	
A.2	Write, read, and evaluate expressions in which letters stand for numbers.
B.5	Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
B.6	Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
B.7	Solve real-world and mathematical problems by wiring and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.
C.9	Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.
B.3	Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of

	operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.
B.4	Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

Instructional Focus

Unit Enduring Understandings

- Comparing relationships can deepen understanding.
- Patterns can be used to describe and predict phenomenon.
- Mathematics can be explained in the oral or written form
- One representation may sometimes be more helpful than another; and, used together, multiple representations give a fuller understanding of a problem
- Everyday objects have a variety of attributes, each of which can be measured in many ways
- What we measure affects how we measure it
- Mathematics is based on patterns, relationships, and a defined set of rules that interconnect and explain all mathematical concepts and natural phenomena
- Spatial sense and various geometric terms, concepts, and properties are used to model, identify, interpret, and describe relationships as they exist in the world; the terms, properties, and concepts chosen are based on the situation and what is to be communicated.
- Geometric properties can be used to construct geometric figures.
- Geometric relationships provide a means to solve real-life problems

Unit Essential Questions

- How can relationships be compared?
- How does patterning help with prediction?
- What is the best way to communicate mathematically?
- How can the collection, organization, interpretation, and display of data be used to answer questions?
- What is the best way to measure?
- How can measurements be used to solve problems?
- How can change be best represented mathematically?
- How can spatial relationships be described by careful use of geometric language?

Objectives

Students will know:

- the vocabulary associated with the coordinate plane;
- the values on the axes increase the farther something is from the origin;
- the meaning of the coordinates of an ordered pair;
- independent and dependent variables from a graph and table;
- strategies for determining a rate of change;
- the meaning and impact of slope and a y- intercept;
- strategies for identifying and applying ratios, unit rates, and proportions;
- strategies for describing a relationship between two variables;
- the connection between constant rates of change, slope, and the impact on an equation;
- the common solution of two linear equations is the point of intersection when the equations are graphed.

Students will be able to:

- Identify the variables in a given situation;
- Interpret points on a graph in relation to their position on the graph;

- Recognize that the values on the x-axis increase from left to right on a coordinate graph;
- Recognize that the values on the y-axis increase from bottom to top on a coordinate graph;
- Identify and explain the relationship between the independent and dependent variables represented on a graph, table or equation;
- Create a graph that represents collected data;
- Identify and compare the rates of change in varying situations (include constant rates of change);
- Describe change and rates of change between variables represented on a graph, table or equation;
- Identify the y-intercept on a graph and/or equation and interpret how it impacts a situation;
- Identify ratios, unit rates, and proportions;
- Use ratios, unit rates, and proportions to complete tables and problem solve;
- Describe relationships by using graphs, tables, and equations for explicit and recursive rules and use the relationship to predict;
- Identify a greater rate of change produces a steeper slope and that a smaller rate of change produces a slope that is less steep;
- Analyze the effect of rates of change, slope, and their impact on an equation;
- Compare the point of intersection of two linear graphs to the common solution when two linear equations are set equal to each other;
- Apply all skills to solve real-world problems

Resources

Core Text: Project M3- Mentoring
Mathematical Minds

Suggested Resources/Activities:

- Have students use laptops/iPads to graph and analyze linear functions on Desmos.
- https://www.flourishkh.com
- Think Beyond and Think Deeply Cards
- Hint Cards for students who need support
- www.schoology.com journal responses