# PISCATAWAY TOWNSHIP SCHOOLS 

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## Algebra 2

## Content Area: Mathematics <br> Grade Span: 9-12

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## COURSE OVERVIEW

## Description

Algebra 2 builds on concepts mastered in Algebra 1 and Geometry and prepares students for course work in Precalculus, Calculus, and Statistics courses. Students will extend their understandings of linear, quadratic, and exponential functions to new scenarios and deeper levels of complexity and rigor. Higher-order polynomial, logarithmic, radical, and rational functions will be introduced and comparisons between the function types will be a central theme. Students will also extend their understanding of triangle trigonometry to trigonometric functions on the coordinate plane and will explore the unit circle in preparation for further discussion in Precalculus. Students will also examine statistical modeling and sampling during the course. Modeling and problem solving will be embedded throughout each unit to ensure that students can transfer their algebraic understandings to real-life scenarios.

## Goals

In addition to the content standards, skills, and concepts set forth, this course also seeks to meet the Standards for Mathematical Practice. These practices include generally applied best practices for learning mathematics, such as understanding the nature of proof and having a productive disposition towards the subject, and are not tied to a particular set of content. These skills are applicable beyond a student's study of mathematics.

The eight Standards for Mathematical Practice are outlined below:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

| Scope and Sequence |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Unit | Topic | Length (Approx 90 Blocks) <br> Regular : Honors |  |  |
| Unit 0 | Algebra Essentials | $5: 0$ |  |  |
| Unit 1 | Linear and Absolute Value Functions | $8: 10$ |  |  |
| Unit 2 | Quadratic Functions | $10: 10$ |  |  |
| Unit 3 | Higher Order Polynomials | $11: 12$ |  |  |
| Unit 4 | Powers, Roots, and Radical Functions | $11: 12$ |  |  |
| Unit 5 | Trigonometric Functions | $11: 11$ |  |  |
| Unit 6 | Exponential \& Logarithmic Functions | $11: 11$ |  |  |
| Unit 7 | Rational Functions | $11: 12$ |  |  |
| Unit 8 |  |  |  |  |
| Statistics |  |  |  | $5: 5$ |
| Core Text: Algebra 2: Big Ideas Learning (2022 Edition by: Ron Larson, Laurie Boswell) <br> Suggested Resources: Desmos, Delta, <br> Math, Kuta Software |  |  |  |  |
|  |  |  |  |  |

## UNIT O: Algebra Essentials

## Summary and Rationale

This introductory unit will serve as a bridge of skills for students entering academic Algebra 2 . The focus will be on skills including solving equations and simplifying expressions. Assessment will be used to gauge student competency levels and instruction will be differentiated based on those needs regarding the topics highlighted below. These skills are intertwined throughout the duration of the Algebra 2 curriculum.

## Recommended Pacing <br> Academic: $5 \quad$ Honors: 0 <br> State Standards

| Standard A-REI Reasoning with Equations and Inequalities |  |
| :---: | :---: |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 6 | Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. |
| Standard F-IF Intepreting Functons |  |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 4 | For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship |
| 7a | Graph linear and quadratic functions and show intercepts, maxima, and minima |
| Instructional Focus |  |
| Unit Enduring Understandings |  |
|  | uations may be used as models to solve mathematical and real-world problems. <br> al-world problems may be represented by the formation and solution of linear equations. ationships can be analyzed to make predictions and draw conclusions. <br> terns and relationships can be represented graphically, numerically, symbolically, or verbally |
| Unit Essential Questions |  |
|  | $w$ can we create an equation for a given situation? <br> w does the way we represent a solution help us understand its meaning? <br> $w$ can change be best represented mathematically? <br> at makes an algebraic algorithm both effective and efficient? |

## Objectives

## Students will know:

- The difference between simplifying expressions and solving equations
- Order of Operations
- Basic shapes of linear functions
- $\quad f(x)$ is a dependent variable determined by the value of an independent variable, $x .[y=f(x)]$
- Point slope form is a manipulation of the slope formula
- How to write an equation of a linear function given critical attributes of the function
- The solution to a system of equations represents the intersection of the graphs.
- The difference between standard form and slope intercept form of a line
- The $y$-intercept occurs when $x=0$ and the $x$-intercept occurs when $y=0$.

Vocabulary: x-intercept, $y$-intercept, slope, slope-intercept form

## Students will be able to:

- Simplify using order of operations
- Write an equation of a line given two points using point slope form
- Write an equation of a line given the slope and a point
- Analysis of a linear function using equation, table, and graph
- Identify the input and output of a function in proper function notation
- Evaluate an output for a function given a specific input
- Connect linear equations to arithmetic sequences
- Convert from standard form of a line to slope intercept form of a line
- Given any graph, state $x$ and $y$ intercepts as ordered pairs
- Solve equations
- Solve a system graphically
- Write a linear models that depicts a real world scenario


## Resources

Core Text: Algebra 2: Big Ideas Learning (2022 Edition by: Ron Larson, Laurie Boswell)
Suggested Resources: Desmos, Delta Math, Kuta Software

## UNIT 1: Linear \& Absolute Value

## Summary and Rationale

The introductory unit builds on understandings from Algebra 1 and its focus on functions and transformations. Linear functions will be reintroduced as part of the set of polynomial graphs and will be explored in a number of equation forms. Absolute value functions, piecewise functions, and linear inequalities will also be explored. Explorations in this unit set the standard for the graph analysis that will occur during the course, including analysis of the meaning of a solution, an understanding of domain and range, and the effects of transformations on each function explored. Modeling and problem solving will be embedded throughout the unit to ensure that students can transfer their algebraic understandings to real-life scenarios.

Honors only: This unit will also explore the study of number sets, Venn Diagrams, and the conjunction and disjunction of solution sets.

## Recommended Pacing

Academic: 8
Honors: 10

## State Standards

| Standard A-CED Creating Equations |  |
| :--- | :--- |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 1 | Create equations and inequalities in one variable and use them to solve problems. Include equations <br> arising from linear and quadratic functions, and simple rational and exponential functions. |
| 2 | Create equations in two or more variables to represent relationships between quantities; graph equations <br> on coordinate axes with labels and scales. |
| 3 | Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and <br> interpret solutions as viable or nonviable options in a modeling context. For example, represent <br> inequalities describing nutritional and cost constraints on combinations of different foods. |

## Standard A-REI Reasoning with Equations and Inequalities

## CPI \# $\quad$ Cumulative Progress Indicator (CPI)

| 1 | Explain each step in solving a simple equation as following from the equality of numbers asserted at the |
| :--- | :--- | previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
11 Explain why the $x$-coordinates of the points where the graphs of the equations $y=f(x)$ and $y=g(x)$ intersect are the solutions of the equation $f(x)=g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, or absolute value functions.
Standard A-SSE Reasoning with Equations and Inequalities

| CPI \# | Cumulative Progress Indicator (CPI) |
| :--- | :--- |
| 1 | Interpret expressions that represent a quantity in terms of its context. |
| 2 | Use the structure of an expression to identify ways to rewrite it. |
| 3 | Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity <br> represented by the expression. |

## Standard F-IF Reasoning with Equations and Inequalities

| CPI \# | Cumulative Progress Indicator (CPI) |
| :--- | :--- |
| 2 | Use function notation, evaluate functions for inputs in their domains, and interpret statements that use <br> function notation in terms of a context. |
| 4 | Compare properties of two functions each represented in a different way (algebraically, graphically, <br> numerically in tables, or by verbal descriptions). |
| 7 a | Graph linear and quadratic functions and show intercepts, maxima, and minima. |
| 7 b | Graph piecewise-defined functions, including step functions and absolute value functions. |
| 9 | Compare properties of two functions each represented in a different way (algebraically, graphically, <br> numerically in tables, or by verbal descriptions). |

## Standard N-RN The Real Number System

| 3 | Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and <br> an irrational number is irrational; and that the product of a nonzero rational number and an irrational <br> number is irrational. |
| :--- | :--- |

## Instructional Focus

## Unit Enduring Understandings

- The symbolic language of algebra is used to communicate and generalize the patterns in mathematics
- Algebraic representation can be used to generalize patterns and relationships
- Patterns and relationships can be represented graphically, numerically, symbolically, or verbally
- Mathematical models can be used to describe and quantify physical relationships
- Physical models can be used to clarify mathematical relationships
- Algebraic and numeric procedures are interconnected and build on one another to produce a coherent whole
- Reasoning and/or proof can be used to verify or refute conjectures or theorems in algebra


## Unit Essential Questions

- How can change be best represented mathematically?
- How can patterns, relations, and functions be used as tools to best describe and help explain real-life situations?
- How are patterns of change related to the behavior of functions?
- How can we use mathematical models to describe physical relationships?
- How can we use physical models to clarify mathematical relationships?
- What makes an algebraic algorithm both effective and efficient?


## Objectives

## Students will know:

- Basic shapes of linear, absolute value, and quadratic parent graphs
- A piecewise function is the union of 2 or more sub-functions, each having its own restricted domain
- The differences in domain and range
- $\quad f(x)$ is a dependent variable determined by the value of an independent variable, $x$. $[y=f(x)]$
- A series of transformations in functional notation for absolute value graphs
- How to write an equation of a function given critical attributes of the function
- Systems of nonlinear equations can have no solution, one solution, or multiple solutions
- Real world scenarios may create mathematical restraints in a problem
- The solution to a system of equations represents the intersection of the graphs.
- Differences in conjunctions and disjunctions in relation to compound inequalities.
- **Honors only** How to find the solutions of nonlinear and 3-variable systems

Vocabulary: domain, range, inclusive, exclusive, intersection, union, compound inequality, absolute value, $x$-intercept, $y$-intercept, set-builder notation, interval notation, axis of symmetry, piecewise function, vertical/horizontal stretch/compression/shift

## Students will be able to:

- Write an equation of a line given two points using point slope form (**In Unit 0 for Academic**)
- Write an equation of a line given the slope and a point ( ${ }^{* *}$ In Unit 0 for Academic ${ }^{* *}$ )
- Classify a function given a sketch of a graph
- Identify the shape of a graph given the equation of a function
- Graph the set of solutions to an equation, an inequality, an absolute value function, and a piecewise function
- Analyze graphs of linear, absolute value, piecewise, linear inequalities
- **Honors only** Graph and analyze step functions
- Determine when a graph represents a function using the vertical line test
- Identify the domain and range of a given function using interval notation
- ${ }^{* *}$ Honors only** Identify sets of numbers and their application in real world problems (real numbers versus integers etc.)
- ${ }^{* *}$ Honors only** Identify the domain and range of a given function using interval and set builder notation
- Identify the input and output of a function in proper function notation
- Compare $f(x)$ to $f(x)+k, k f(x), f(k x)$, and $f(x+k)$ for absolute value functions
- **Honors only** Compare $f(x)$ to $f(x)+k, k f(x), f(k x)$, and $f(x+k)$ for various functions
- Evaluate an output for a function given a specific input
- Identify a type of relation given a table of values
- ${ }^{* *}$ Honors only** Review solving absolute value equations, inequalities, special cases (i.e, $x<2$ and $x<3$, $x<2$ or $x<3$ )
- Rewrite an absolute value inequality into two separate single-variable cases
- Use interval notation to express solutions of absolute value inequalities
- Solve systems of linear and absolute value equations by graphing
- Solve systems of linear equations algebraically
- Solve compound inequalities to discuss conjunctions and disjunctions
- **Honors only** Translate a conjunction and disjunction statement using $\cap$ or $U$
- **Honors only** Solve nonlinear and 3 -variable systems
- **Honors only** Use the "calc-intersect" function on a graphing calculator to find points of intersection for non-linear systems


## Resources

Core Text: Algebra 2: Big Ideas Learning (2022 Edition by: Ron Larson, Laurie Boswell)

Suggested Resources: Desmos, Delta Math, Kuta Software

## UNIT 2: Quadratic Functions

## Summary and Rationale

This unit continues Algebra 1 understandings of quadratic functions and their transformations within the context of the set of all polynomial functions. Extensive comparison of the three quadratic forms and their solutions will elucidate the algebraic relationships between the forms and will provide an understanding of the value of each equation in graph analysis. Students will also be introduced to the set of complex numbers and will explore the relationship between quadratic graphs and complex solutions. This unit will also include basic introductions of end behavior, rate of change, and limits. Modeling and problem solving will be embedded throughout the unit to ensure that students can transfer their algebraic understandings to real-life scenarios.

## Recommended Pacing

Academic: $10 \quad$ Honors:10

## State Standards

## Standard A-APR Arithmetic with Polynomials and Rational Expressions

| CPI \# | Cumulative Progress Indicator (CPI) |
| :--- | :--- |
| 3 | Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a <br> rough graph of the function defined by the polynomial. |
| 4 | Prove polynomial identities and use them to describe numerical relationships. |
| Standard A-CED Creating Equations |  |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 1 | Create equations and inequalities in one variable and use them to solve problems. Include equations <br> arising from linear and quadratic functions, and simple rational and exponential functions. |
| 2 | Create equations in two or more variables to represent relationships between quantities; graph equations <br> on coordinate axes with labels and scales. |
| 3 | Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and <br> interpret solutions as viable or nonviable options in a modeling context. For example, represent <br> inequalities describing nutritional and cost constraints on combinations of different foods. |

## Standard A-REI Reasoning with Equations and Inequalities

## CPI \# $\quad$ Cumulative Progress Indicator (CPI)

4a Use the method of completing the square to transform any quadratic equation in $x$ into an equation of the form $(x-p)^{2}=q$ that has the same solutions. Derive the quadratic formula from this form.
4b Solve quadratic equations by inspection (e.g., for $x^{2}=49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm b i$ for real numbers $a$ and $b$.
11 Explain why the $x$-coordinates of the points where the graphs of the equations $y=f(x)$ and $y=g(x)$ intersect are the solutions of the equation $f(x)=g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are quadratic functions.

## Standard A-SSE Reasoning with Equations and Inequalities

| CPI \# | Cumulative Progress Indicator (CPI) |
| :--- | :--- |
| 1 | Interpret expressions that represent a quantity in terms of its context. |
| 1 a | Interpret parts of an expression, such as terms, factors, and coefficients. |
| 2 | Use the structure of an expression to identify ways to rewrite it. |


| 3 | Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. |
| :---: | :---: |
| 3.A | Factor a quadratic expression to reveal the zeros of the function it defines. |
| 3.8 | Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. |
| Standard F-IF Reasoning with Equations and Inequalities |  |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 2 | Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. |
| 4 | Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). |
| 5 | Graph linear and quadratic functions and show intercepts, maxima, and minima. |
| 8 | Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. |
| 8 a | Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. |
| Standard N-CN The Complex Number System |  |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 1 | Know there is a complex number $i$ such that $i^{2}=-1$, and every complex number has the form $a+b i$ with $a$ and $b$ real. |
| 2 | Use the relation $i^{2}=-1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. |
| 3 | (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. |
| 7 | Solve quadratic equations with real coefficients that have complex solutions. |
| 8 | (+) Extend polynomial identities to the complex numbers. For example, rewrite $x^{2}+4$ as $(x+2 i)(x-2 i)$. |
| 9 | $(+)$ Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. |
|  | Instructional Focus |
| Unit Enduring Understandings |  |
| - There are mathematical patterns everywhere. <br> - To change one parameter of a function will change the look of the image. <br> - The zeroes of a function are the roots or $x$-intercepts <br> - There is more than one way to find the roots of a quadratic function <br> - Real life scenarios can be modeled with quadratic equations. |  |
| Unit Essential Questions |  |
| - What are the critical attributes of a given function? <br> - How do you transform a graph? <br> - How can you restrict a function? <br> - What will you expect a family of functions to look like? <br> - How are the $x$-intercepts of a quadratic function related to the solutions of a quadratic equation? <br> - How many ways can you find the roots of a quadratic function? |  |
| Objectives |  |
| Students will know: |  |

- How to write an equation of a function given critical attributes of the function.
- An algorithm for factoring quadratic functions.
- The quadratic formula can be derived by completing the square on a standard form quadratic equation
- The definition of $i$ and the pattern for the powers of $i$
- A complex number is the sum of a real part and imaginary part. $(a+b i)$
- Conjugates are used when dividing complex expressions
- Complex conjugate solutions to quadratic equations always come in pairs.
- A complex solution to a quadratic equation implies there is no $x$-intercept to the related quadratic function.
- A quadratic function can be written in vertex form, standard form, and intercept form.
- How to find critical attributes of a parabola given a specific form of a quadratic function.
- The $y$-intercept occurs when $x=0$ and the $x$-intercept occurs when $y=0$.
- The roots (solutions) to a quadratic equation are the zeros (x-intercepts) of the related quadratic function.
- The vertex always represents a minimum or a maximum
- All even functions are symmetric over the $y$-axis.
- Quadratic functions are only even if the graph is limited to transformations that retain $y$-axis symmetry
- Basic shapes of linear, absolute value, and quadratic parent graphs

Vocabulary: domain, range, inclusive, exclusive, $x$-intercept (zeros), $y$-intercept, set-builder notation, interval notation, axis of symmetry, quadratic functions, vertex, complex number system, rational, irrational, imaginary number, quadratic formula, discriminant, completing the square, factor, roots, zeros, vertical/horizontal stretch/compression/shift.

## Students will be able to:

- Create equations to represent relationships and to solve real world problems
- Factor to find roots of quadratic functions
- Identify difference of squares and perfect square trinomials
- Use quadratic techniques to connect $(x+3 i)(x-3 i)$ and $x^{2}+9$
- Define i and simplify powers of i
- Connect complex solutions of quadratic equations to imaginary zeros of quadratic functions (through quadratic formula and completing the square)
- Solve the same quadratic equation four ways: factoring, quadratic formula, completing the square, looking at the related graph
- Identify critical attributes of a graph from the three forms of a quadratic function
- Transition from one form of a quadratic to another
- Find $x$ - and $y$-intercepts of a function algebraically and graphically
- Write an equation of a quadratic function given a vertex and a point on the parabola
- Graph systems of quadratics, absolute value, and linear functions
- Graph analysis of quadratic functions
- Compare $f(x)$ to $f(x)+k, k f(x), f(k x)$, and $f(x+k)$ for quadratic functions
- Identify the domain and range of all functions covered
- Perform operations (add, subtract, and multiply) with complex numbers.
- **Honors only** Divide complex numbers
- Write an expression as a complex number in standard form and identify the real and imaginary components
- Use the discriminant to identify the type and number of solutions to a given quadratic equation.
- Complete the square in order to rewrite a standard form equation (where a=1) into vertex form.
- **Honors only** Complete the square in order to rewrite a standard form equation (where a may or may not equal 1) into vertex form.
- Find the $x$-coordinate of the vertex of a parabola using $x=-b / 2 a$ and by finding the midpoint of the roots.
- Use a graphing technology to see the amount of intersections for non-linear systems in two variables.
- Use a graphing technology to see the connection between parabolas that do not touch the x - axis and its related quadratic equation that has imaginary solutions
- **Honors only** Solve quadratic inequalities graphically and algebraically
- Solve projectile motion modeling problems
- **Honors only** Solve projectile motion, max profit, max area modeling problems


## Resources

Core Text: Algebra 2: Big Ideas Learning (2022 Edition by: Ron Larson, Laurie Boswell)
Suggested Resources: Desmos, Delta Math, Kuta Software

## UNIT 3: Higher Order Polynomials

## Summary and Rationale

This unit extends the understandings of linear and quadratic functions to higher-order polynomials and their transformations. In the context of these functions, students will explore end behavior, the relationship between roots and $x$-intercepts, multiplicity of roots, and more rigorous solving methods. The notion of odd and even functions will be discussed as students begin to make high-level connections regarding the nature of polynomial functions. Students will continue to explore the relationship between complex solutions and polynomial graphs along with rate of change and limits. Modeling and problem solving will be embedded throughout the unit to ensure that students can transfer their algebraic understandings to real-life scenarios.

## Recommended Pacing

Academic: $11 \quad$ Honors: 12

## State Standards

## Standard A-APR Arithmetic with Polynomials and Rational Expressions

| CPI \# | Cumulative Progress Indicator (CPI) |
| :--- | :--- |
| 1 | Understand that polynomials form a system analogous to the integers, namely, they are closed under the <br> operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. |
| 2 | Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number $a$, the remainder on <br> division by $x-a$ is $p(a)$, so $p(a)=0$ if and only if $(x-a)$ is a factor of $p(x)$. |
| 3 | Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a <br> rough graph of the function defined by the polynomial. |
| 4 | Prove polynomial identities and use them to describe numerical relationships. |
| Standard F-IF Reasoning with Equations and Inequalities |  |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 5 | Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end <br> behavior. |
| 8 | Write a function defined by an expression in different but equivalent forms to reveal and explain different <br> properties of the function. |

## Standard N-CN The Complex Number System

| CPI \# | Cumulative Progress Indicator (CPI) |
| :--- | :--- |
| 8 | Extend polynomial identities to the complex numbers. For example, rewrite $x^{2}+4$ as $(x+2 i)(x-2 i)$. |
| 9 | Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. |

## Instructional Focus

## Unit Enduring Understandings

- Roots and vertices of functions have real-world meaning.
- Equations that model real-world data allow you to make predictions about the future.
- Polynomial shapes can be predicted.
- Graphs allow you to estimate solutions.


## Unit Essential Questions

- What type of equation would model specific real-world data?
- Why is the vertex an important point of a parabola?
- Why is a remainder of zero significant when dividing?
- How can factoring help graph a polynomial function?


## - Can you find roots of a graph without factoring?

## Objectives

## Students will know:

- The differences between the end behaviors of even and odd degree polynomial
- Domain and range for polynomial functions
- Proper notation for end behavior
- The general shapes of several polynomial functions
- A power function is a function: $y=x^{n}$ where n is any real constant
- Complex conjugate roots always come in pairs
- How to factor the sum and difference of cubes,
- The trinomial factor for the difference of cubes model is always a pair of complex conjugate solutions.
- The multiplicity of roots in relation to the behavior of a polynomial graph (crossing vs. bouncing on the $x$-axis)
- Polynomials written in factored form infer roots of the related function
- For all even functions $f(-x)=f(x)$ and there is always y -axis symmetry
- For all odd functions $f(-x)=-f(x)$ and there is always 180 degree rotational symmetry about the origin
- Some functions are neither odd nor even
- There are various methods of factoring based on the amount of terms in a polynomial expression.
- The appropriate uses of both long division and synthetic division
- **Honors only** Sign changes from Descartes' rule of signs help indicate the amount of real roots to a polynomial equation.
- A polynomial equation of degree n has exactly n roots in the set of complex numbers and up to $n-1$ turns
- **Honors only** The factors of the constant divided by the factors of the leading coefficient provide a set of possible rational zeros to a polynomial equation
- The factors of polynomial equations (roots) connect to the x-intercepts (zeros) of a polynomial function.
- The fundamental theorem of algebra

Vocabulary: depressed equation, domain, range, $x$-intercept (zeros), solutions (roots), $y$-intercept, interval notation, axis of symmetry, quadratic, cubic, quartic, power function, polynomial function, vertex, relative maximum, relative minimum, quadratic formula, end-behavior, long division, synthetic division, operations with polynomials, laws of exponents, degree, odd function, even function, end behavior

## Students will be able to:

- Perform operations with polynomials
- Identify end behaviors of a polynomial function (odd vs. even degree) and the behavior at any x-intercept (multiplicity of roots)
- Use various quadratic factoring techniques on higher order polynomials to find zeroes - i.e.: factor by grouping and factor using u-substitution
- Identify and factor difference of squares, difference and sum of cubes, and perfect square trinomials
- Use synthetic and long division to find roots and depressed equations
- **Honors only** Use the Rational Zero Theorem and Descartes' rule of signs
- Connect x-intercepts (zeros) of functions to factors of equations (roots)
- Write an equation of a polynomial functions given some or all of its roots
- Write an equation given sketch of a graph
- Compare $f(x)$ to $f(x)+k, k f(x), f(k x)$, and $f(x+k)$ for power functions
- Examine critical attributes (domain, range, intercepts, intervals of increasing/decreasing) of all functions covered
- Identify polynomial functions as odd, even, or neither
- Use proper notation to describe the end behaviors of a polynomial function. $[f(x) \rightarrow+\infty$ as $x \rightarrow+\infty]$
- Identify the number and type of zeros to a polynomial function
- Find the zeros and sketch a polynomial function given the linear factors of a polynomial expression
- Graph polynomial functions using critical attributes
- Hand sketch power functions quickly and accurately - connect end behavior to graphing higher order polynomial functions
- Use graphing technology to estimate relative maximums and minimums as well as zeros of higher order polynomial functions by evaluating the "table" and "table set" options
- **Honors only** Solve polynomial inequalities both algebraically and graphically
- **Honors only** Solve modeling problems including but not limited to volume applications


## Resources

Core Text: Algebra 2: Big Ideas Learning (2022 Edition by: Ron Larson, Laurie Boswell)

Suggested Resources: Desmos, Delta Math, Kuta Software

## UNIT 4: Powers, Roots, Radical Functions

## Summary and Rationale

This unit examines the analysis of radical functions and transformations on those graphs. Students will review the properties of exponents as discussed in Algebra 1 and will extend their understanding to different rational exponents with more challenging applications. Discussions on domain and range become more complex during this unit as functions begin to take on new shapes and follow different algebraic rules. Continued exploration of end behavior, the relationship between roots and $x$-intercepts, odd and even functions, and limits occur during this unit. Additional topics include inverse functions and composition of two functions. Modeling and problem solving will be embedded throughout the unit to ensure that students can transfer their algebraic understandings to real-life scenarios.

## Recommended Pacing

## Academic: 11

Honors: 12

## State Standards

| Standard A-CED Creating Equations |  |
| :--- | :--- |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 1 | Create equations and inequalities in one variable and use them to solve problems. |
| 2 | Create equations in two or more variables to represent relationships between quantities; graph equations <br> on coordinate axes with labels and scales. |
| 3 | Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and <br> interpret solutions as viable or nonviable options in a modeling context. |
| 4 | Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. |
| Standard A-REI Reasoning with Equations and Inequalities |  |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 2 | Solve simple rational and radical equations in one variable, and give examples showing how extraneous <br> solutions may arise. |
| 10 | Understand that the graph of an equation in two variables is the set of all its solutions plotted in the <br> coordinate plane, often forming a curve (which could be a line). |
| Standard A-SSE Reasoning with Equations and Inequalities |  |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 1a | Interpret parts of an expression, such as terms, factors, and coefficients. |
| 2 | Use the structure of an expression to identify ways to rewrite it. |
| 3 | Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity <br> represented by the expression. |
| Standard F-BF Building Functions |  |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 4 | Find inverse functions. |
| $4 a$ | Solve an equation of the form $f(x)=c$ for a simple function $f$ that has an inverse and write an expression <br> for the inverse. For example, $f(x)=2 x^{3}$ or $f(x)=(x+1) /(x-1)$ for $x \neq 1$. <br> $4 b$ <br> (+) Verify by composition that one function is the inverse of another. <br> $4 c$ <br> (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. Produce an invertible function from a non-invertible function by restricting the domain. |


| Standard F-IF Reasoning with Equations and Inequalities |  |
| :---: | :---: |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 6 | Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. |
| 7b | Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. |
| 8 | Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. |
| 9 | Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). |
| Standard N-RN The Real Number System |  |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 1 | Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. |
| 2 | Rewrite expressions involving radicals and rational exponents using the properties of exponents. |
| Instructional Focus |  |
| Unit Enduring Understandings |  |
| - Analyzing trends predict behavior. <br> - There are many ways to solve a problem. <br> - Procedures can be reversed. <br> - Make sense of problems and persevere in solving them. |  |
| Unit Essential Questions |  |
| - How does the $x$-intercepts of the graph relate to the solutions of the equation? <br> - How do you find an inverse relation of a given function? <br> - What do the graphs of square root and cube functions look like? <br> - Why is it necessary to check every apparent solution of a radical equation in the original equation? <br> - In what ways can powers, roots, and radicals be used in solving real world problems? |  |
| Objectives |  |
| Students will know: <br> - The properties of exponents. <br> - Rational exponents can be written in radical notation <br> - The laws of exponents applies to the properties of rational exponents <br> - A fraction is not considered to be in simplest form if it contains a radical expression in its denominator <br> - To solve a radical equation you need to isolate then eliminate the radicals or rational exponents and obtain a polynomial equation to solve using quadratic techniques <br> - Radical equations may have extraneous solutions <br> - The general shape of the square and cube root graph <br> - Proper notation for domain, range and end-behavior, intervals of increasing/decreasing <br> - The domain and range of a radical function may have restrictions <br> - The series of transformations to the parent functions of square and cube root graphs <br> - For all even functions $f(-x)=f(x)$ and there is always y -axis symmetry <br> - For all odd functions $f(-x)=-f(x)$ and there is always 180 degree rotational symmetry about the origin <br> - Function operations and how they affect the domain of the function <br> - Composition of radical functions will change the domain and range of the final result <br> - The definition of inverses <br> - The ordered pair solutions for one equation are switched for the solutions of the inverse <br> - Composition of functions is an algebraic approach to determine if the functions are inverses |  |

Vocabulary: domain, range, $x$-intercept (zeros), solutions (roots), extraneous, nth root of, $y$-intercept, set-builder notation, interval notation, end-behavior, intervals of increasing/decreasing, radical, laws of exponents, square root, cube root, inverse, composition, index, function, conjugate, horizontal and vertical line test
**Honors only** one-to-one, even/odd functions

## Students will be able to:

- Simplify expressions involving rational powers and variables using the properties of exponents
- Simplify radical expressions (square root, cube root, 4th root)
- **Honors only**: Explore when variables are not assumed to be positive
- Convert an expression from radical notation to exponential notation and vice vers (5.1)
- Perform operations to simplify radical expressions
- Rationalize radical expression denominators using the conjugate
- **Honors only** Use properties of exponents to correctly simplify radical expressions (multiplying to make exponent of denominator an integer value)
- Solve radical and rational equations and check for extraneous solutions
- **Honors only**: Solve radical rational inequalities
- Graph square root and cube root functions
- Perform transformations on square root and cube root functions
- Reflections, translations, stretches/shrinks (compressions)
- Compare $f(x)$ to $f(x)+k, k f(x), f(k x), f(x+k)$
- Identify end behaviors of a radical function
- Identify intervals of increasing/decreasing of a radical function
- Identify domain and range
- Identify odd and even functions both graphically and algebraically
- Simplify and evaluate the composition of functions
- $\quad(f+g)(x),(f-g)(x),(f g)(x),(f / g)(x), f(g(x))$
- Use composition of functions to portray real life situations
- **Honors only** Graph the composition of functions
- Graphically and algebraically find the inverse of relations/functions
- Determine values/ordered pairs of a inverse function given a graph or table
- Utilize the results of the vertical and horizontal line test to determine function status
- **Honors only** Discuss one-to-one functions
- **Honors only** Model real life scenarios and discuss the meaning of $f^{\wedge}-1(x)$
- **Honors only** Algebraically prove two relations are inverses using $f(g(x))=x, g(f(x))=x$


## Resources

Core Text: Algebra 2: Big Ideas Learning (2022 Edition by: Ron Larson, Laurie Boswell)
Suggested Resources: Desmos, Delta Math, Kuta Software

## UNIT 5: Trigonometric Functions

## Summary and Rationale

This unit extends students' understanding of the basic trigonometric functions used with triangles (sine, cosine, and tangent) to explore their operation on a coordinate plane. Students will first explore the unit circle, reference angles, and coterminal angles, and will then transfer that understanding to build functions on the coordinate plane. Graph analysis of amplitude, period, and phase shift will be explored through an examination of the transformations on these functions and discussion of domain and range will be embedded throughout. Modeling and problem solving will be embedded throughout the unit to ensure that students can transfer their algebraic understandings to real-life scenarios.

## Recommended Pacing

## Academic: $11 \quad$ Honors: 11

## State Standards

## Standard F-TF Trigonometric Functions

| CPI \# | Cumulative Progress Indicator (CPI) |
| :--- | :--- |
| 1 | Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. |
| 2 | Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all <br> real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. |
| 3 | (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi / 3, \pi / 4$ and <br> $\pi / 6$, and use the unit circle to express the values of sine, cosine, and tangent for $x, \pi+x$, and $2 \pi-x$ in <br> terms of their values for $x$, where $x$ is any real number. |
| 4 | $(+)$ Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. |
| 5 | Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and <br> midline. |
| 6 | (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or <br> always decreasing allows its inverse to be constructed. |
| 7 | (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the <br> solutions using technology, and interpret them in terms of the context. |
| 8 | Prove the Pythagorean identity $\sin ^{2}(\Theta)+\cos (\Theta)=1$ and use it to and use it to find $\sin (\theta), \cos (\theta)$, or <br> tan $(\theta)$ given $\sin (\theta), \cos (\theta)$, or $\tan (\theta)$ and the quadrant of the angle. |

Standard G-SRT Similarity, Right Triangles, and Trigonometry
CPI \# $\quad$ Cumulative Progress Indicator (CPI)
1 Us

Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

## Instructional Focus

## Unit Enduring Understandings

- There are six trigonometric ratio functions related to a right triangle.
- Trig functions allow you to solve any right triangle.
- Real world phenomena can be modeled using trig functions.
- The relationship between circumference and radians.
- One wave of a sine or cosine graph is equivalent to one rotation on the unit circle


## Unit Essential Questions

- How can you use trig ratios to solve problems that model triangles?
- How do you evaluate the six trig functions for any given reference angle measure without a calculator?
- How can your knowledge of the unit circle help you graph trig functions?


## Objectives

## Students will know:

- There are six trigonometric functions related to a triangle
- Trig functions allow you to solve any right triangle
- The significance of the coordinates along the unit circle and how they relate to the six trig functions.
- There are two units of angle measurement: degrees and radians
- The degree measure of an angle is the rotation from the initial side to the terminal side in a counter-clockwise rotation, about the origin, from the positive $x$-axis
- The radian measure of an angle is the length of the arc on the unit circle.
- One complete rotation is 360 degrees or $2 \pi$ radians.
- On the unit circle, $x=\cos (\theta), y=\sin (\theta), y / x=\tan (\theta)$
- The value of a trigonometric ratio is dependent on what quadrant the angles terminal side is in
- Some functions are neither odd nor even
- For all even functions, $f(-x)=f(x)$ with y -axis symmetry
- For all odd functions, $f(-x)=-f(x)$ with 180 degree rotational symmetry about the origin
- Real world phenomena can be modeled using trig functions
- Periodic functions have a domain and range but an undefined end behavior due to the oscillating graphs
- The basic graph shapes of parent functions sin, cos, tan and understanding of different transformations of those graphs
- **Honors only**: The basic graphs of parent functions sec, csc, cot as well as $\sin ^{-1}, \cos ^{-1}, \tan ^{-1}$

Vocabulary: six trigonometric functions (sine, cosine, tangent, secant, cosecant, cotangent), reference angle, degrees, radians, quadrantal angle, standard position, initial side, terminal side, positive angle (counterclockwise rotation), negative angle (clockwise rotation), coterminal angles, unit circle, oscillating curve, period, amplitude, midline, phase shift

## Students will be able to:

- Use special right triangles to geometrically determine the values of sine, cosine, tangent for $\pi / 3, \pi / 4, \pi / 6$
- Use the six trig ratios to solve right triangles
- Reproduce coordinates along the unit circle using knowledge of reference angles and symmetry
- Evaluate the six trig functions for any angle measure without a calculator
- Work on the xy- coordinate plane
- Draw angles in standard position
- Identify reference angles, coterminal angles, equiv degrees/radians
- Use the unit circle to evaluate the 6 trig functions
- Determine the value of the trig function based on its quadrant
- **Honors only**: Evaluate the trig six functions given a point not on the unit circle
- ${ }^{* *}$ Honors Only**: Problem solve given trigonometric inequality relationships (If $\sin \theta>0$ and $\cos \theta<0$, then...)
- Convert between degrees and radians
- Graph the parent functions: $f(x)=\sin (x), f(x)=\cos (x), f(x)=\tan (x)$
- Compare and graph $f(x)$ to $f(x)+k, k f(x), f(k x)$, and $f(x+k)$ for $\sin , \cos$, tan
- Identify the amplitude, period, phase shift, vertical shift, domain, and range for sine and cosine
- Perform graph analysis of sine, cosine, tangent functions such as naming features, writing the equation of the graph, etc
- $\quad{ }^{* *}$ Honors Only**: Graph the parent functions: $f(x)=\sec (x), f(x)=\csc (x), f(x)=\cot (x)$
- **Honors Only**: Compare $f(x)$ to $f(x)+k, k f(x), f(k x)$, and $f(x+k)$ for sec, csc, cot
- ${ }^{* *}$ Honors Only**: Graph the parent functions: $\sin ^{-1}, \cos ^{-1}, \tan ^{-1}$
- Model and/or interpret real world phenomena using trig functions
- Solve basic trigonometric equations
- ${ }^{* *}$ Honors Only**: Solve trigonometric equations and inequalities with and without quadratic structure


## Resources

Core Text: Algebra 2: Big Ideas Learning (2022 Edition by: Ron Larson, Laurie Boswell)
Suggested Resources: Desmos, Delta Math, Kuta Software

## UNIT 6: Exponential and Logarithmic Functions

## Summary and Rationale

In this unit students explore exponential functions and will apply their understanding of inverse functions to exponential functions in order to understand the concept of a logarithm. Algebraic analysis of the logarithm and its components will allow students to understand its purpose within the course and the construction of its graph. Students will examine the effect of different transformations on exponential and logarithmic graphs They will compare functions and explore the nature of asymptotes, the shape of graphs, and transformations on the resulting function. Students will explore a number of logarithmic bases and will be introduced to the natural log and its connection with Euler's number. Modeling and problem solving will be embedded throughout the unit to ensure that students can transfer their algebraic understandings to real-life scenarios.

## Recommended Pacing

Academic: $11 \quad$ Honors: 11

## State Standards

| Standard A-CED Creating Equations |  |
| :--- | :--- |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 4 | Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. |
| Standard A-REI Reasoning with Equations and Inequalities |  |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 11 | Explain why the $x$-coordinates of the points where the graphs of the equations $y=f(x)$ and $y=g(x)$ <br> intersect are the solutions of the equation $f(x)=g(x)$; find the solutions approximately, e.g. using <br> technology to graph the functions, make tables of values, or find successive approximations. Include cases <br> where $f(x)$ and/or $g(x)$ are exponential functions. |
| Standard A-SSE Reasoning with Equations and Inequalities |  |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 1 | Interpret expressions that represent a quantity in terms of its context. |
| $1 a$ | Interpret parts of an expression, such as terms, factors, and coefficients. |
| 2 | Use the structure of an expression to identify ways to rewrite it. |
| 3 | Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity <br> represented by the expression. |
| $3 c$ | Use the properties of exponents to transform expressions for exponential functions. |
| Standard F-BF Building Functions |  |
| CPI \# | Cumulative Progress Indicator (CPI) |
| $4 c$ | (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. |
| 5 | Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and <br> exponents. |
| Standard F-IF Reasoning with Equations and Inequalities |  |
| CPI \# | Cumulative Progress Indicator (CPI) |
| $7 e$ | Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric <br> functions, showing period, midline, and amplitude. |


| 8 | Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. |
| :---: | :---: |
| 8 b | Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y=(1.02)^{t}, y=(0.97)^{t}, y=(1.01)^{12 t}, y=(1.2)^{t / 10}$, and classify them as representing exponential growth or decay. |
| 9 | Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). |
| Standard F-LE Linear and Exponential Models |  |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 1 | Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. |
| 2 | Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). |
| Standard N-Q Quantities |  |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 1 | Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. |
| 2 | Define appropriate quantities for the purpose of descriptive modeling |
|  | Instructional Focus |
| Unit Enduring Understandings |  |
| $\begin{aligned} & \text { - } \quad \text { An } \\ & \text { - } \\ & \text { - } \\ & \text { - } \\ & \text { - } \end{aligned}$ | alyzing trends predict behavior re are many ways to solve a problem cedures can be reversed preciation and depreciation affect the rate of growth |
| Unit Essential Questions |  |
|  | en there are multiple approaches, how should you choose the best method? <br> understanding a relationship help make a prediction? <br> $w$ do the parts affect the whole? <br> there be more than one correct solution? <br> en is the correct answer not the best solution? <br> I these two things ever be equal? <br> w can you use the graph of an exponential function to sketch the graph of a logarithmic function? <br> y are predictions beneficial? |
| Objectives |  |
| Students will know: <br> - Logarithmic functions and exponential functions are inverses of each other. <br> - Exponential functions grow faster than any other function (studied in this course). <br> - Euler's number (e) is a very important irrational number used in mathematics approximated by 2.718 <br> - Common log means log base 10 <br> - Natural log means log base e <br> - How to use log properties to manipulate log expressions and equations <br> - How to solve exponential and logarithmic equations <br> - **Honors only** How to solve exponential and logarithmic inequalities <br> - ** Honors only** How to utilize multiple properties to approximate logs |  |

- Exponential growth and decay formulas
- Banking compound interest formula and compound continuously formula
- Basic shapes and attributes (proper notation of domain, range, end behavior) of exponential and log parent functions
- A logarithmic function always has a vertical asymptote since its inverse, an Exponential function, always has a horizontal asymptote
- Logarithmic functions only exist for $x>0$.

Vocabulary: exponential growth, exponential decay, growth/decay factor, asymptote, exponential models:
$y=a b^{x}, y=a(1+r)^{t}, y=a(1+r)^{t}, y=P e^{r t}, y=a(1+r / n)^{n t}$, doubling $\mathrm{b}=2$, tripling $\mathrm{b}=3$, halflife, compound interest, values for n (annually, semiannually, quarterly, monthly, weekly, daily, continuously), Euler's number, base, power, logarithm, common log, natural log

## Students will be able to:

- Solve exponential equations by creating like bases
- Rewrite an equation from exponential form to logarithmic form and vice versa
- Use the change of base formula to evaluate logarithms that are not common log or natural log
- Solve logarithmic equations using basic properties of logs
- Solve exponential equations for the exponent using properties of logarithms
- Condense and expand log expressions using log properties
- ${ }^{* *}$ Honors only**: Derive properties of logarithms
- ${ }^{* *}$ Honors only**: Evaluating logs by hand given approximations
- Derive exponential growth and decay equations that model real world situations
- Solve real world situations for time, rate, principal, or future value
- **Honors only** Solve exponential and log inequalities
- Sketch a graph of an exponential and log function and use proper notation to state the function's domain and range.
- Use a vertical asymptote and horizontal asymptotes to connect to restrictions on the domain of a logarithmic function and an exponential function
- Analyze key characteristics for the graphs of log and exponential (domain, range, asymptotes, intervals of increasing/decreasing, intercepts, limits)
- Compare transformations of log and exponential graphs
- Write equations of exponential functions


## Resources

Core Text: Algebra 2: Big Ideas Learning (2022 Edition by: Ron Larson, Laurie Boswell)
Suggested Resources: Desmos, Delta Math, Kuta Software

## UNIT 7: Rational Functions

## Summary and Rationale

In this unit, students will be introduced to rational functions, in which the numerator and denominator are both polynomial expressions. Students will examine algebraic operations on rational expressions and will use those understandings to build the idea of restricted values. In function form, students will transfer their understanding of restrictions to that of the vertical asymptote and will discuss continuous vs. discontinuous functions. Additionally, the differences and similarities between the vertical asymptote (restriction) and the horizontal asymptote (limit) of the function will be discussed. Students will continue to use transformations to show how these functions shift on a coordinate plane and will continue to describe domain and range of the resulting graphs. Modeling and problem solving will be embedded throughout the unit to ensure that students can transfer their algebraic understandings to real-life scenarios.

## Recommended Pacing

## Academic: 11

Honors: 12

## State Standards

| Standard A-APR Arithmetic with Polynomials and Rational Expressions |  |
| :--- | :--- |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 3 | Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a <br> rough graph of the function defined by the polynomial. |
| 6 | Rewrite simple rational expressions in different forms; write $\mathrm{a}(\mathrm{x}) / \mathrm{b}(\mathrm{x})$ in the form $\mathrm{q}(\mathrm{x}) \mathrm{a}+\mathrm{r}(\mathrm{x}) / \mathrm{b}(\mathrm{x})$, where <br> $\mathrm{a}(\mathrm{x}), \mathrm{b}(\mathrm{x}), \mathrm{q}(\mathrm{x})$, and $\mathrm{r}(\mathrm{x})$ are polynomials with the degree of $\mathrm{r}(\mathrm{x})$ less than the degree of $\mathrm{b}(\mathrm{x})$, using <br> inspection, long division, or, for the more complicated examples, a computer algebra system. |
| 7 | Understand that rational expressions form a system analogous to the rational numbers, closed under <br> addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, <br> and divide rational expressions. |
| Standard A-CED Creating Equations |  |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 1 | Create equations and inequalities in one variable and use them to solve problems. |
| 4 | Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. |
| Standard A-REI Reasoning with Equations and Inequalities |  |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 1 | Solve simple rational and radical equations in one variable, and give examples showing how extraneous <br> solutions may arise. |
| Standard A-SSE Seeing Structure in Expressions |  |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 3 | Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity <br> represented by the expression. |
| 3a | Factor a quadratic expression to reveal the zeros of the function it defines. |
| Standard F-BF Building Functions |  |
| CPI \# | Cumulative Progress Indicator (CPI) |
| $1 b$ | Combine standard function types using arithmetic operations. |
| Standard F-IF Reasoning with Equations and Inequalities |  |


| CPI \# | Cumulative Progress Indicator (CPI) |
| :---: | :---: |
| 2 | Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. |
| 7d | (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. |
| Instructional Focus |  |
| Unit Enduring Understandings |  |
| - There are many ways to solve a problem <br> - An equation may not be defined for all $x$ values <br> - Characteristics allow you to make predictions <br> - Analyzing trends predict behavior <br> - Structure allows for patterns and relationships to develop <br> - Exact answers are not always best answers |  |
| Unit Essential Questions |  |
| - Why are there restricted values in rational expressions? <br> - How do you find excluded values of a rational expression? <br> - Why is it helpful to represent rational expressions in terms of their factors? <br> - Can there be more than one correct answer? <br> - How do the values of $\mathrm{a}, \mathrm{h}$ and k affect the asymptotes in $f(x)=\frac{a}{x-h}+k$ ? |  |
| Objectives |  |
| Students will know: <br> - Factoring rational expressions will identify common factors that can be reduced in both a numerator and denominator to get a simplified form <br> - In order to add or subtract rational expressions, a common denominator is needed <br> - Finding the LCD, will make simplifying easier in the end <br> - Dividing two rational expressions is equivalent to multiplying by the reciprocal of the divisor <br> - A complex fraction is the quotient of rational expressions <br> - A rational equation can be solved with multiple methods/strategies (cross multiply, create common denominators, multiple through by the LCD) <br> - Multiplying each side by the LCD can yield extraneous solutions <br> - Proper notation for domain, range, intervals of increasing/decreasing, and end-behavior of rational functions <br> - Asymptotes of rational functions affect the domain/range of the function <br> - End behaviors are related to limits as x approaches infinity or negative infinity <br> - A continuous function can be drawn without lifting your pencil <br> - When a common factor is reduced, that assumed vertical asymptote becomes a hole (point of discontinuity) <br> - Real world phenomena can be modeled with rational equations/functions <br> - Proper forms for direct, inverse, and joint variation <br> - For direct variation, as x increases y increases; for inverse variation, as x increases y decreases |  |
| Vocab <br> fractio <br> Stude <br> - Si <br> - P <br> - S <br> - G <br> - Find | ulary: variation (direct, inverse, joint), rational function, simplest form of rational expression, complex n, extraneous solution, end behavior, point of discontinuity (hole), asymptotes (horizontal, vertical, slant) <br> s will be able to: <br> mplify rational expressions and complex fractions <br> rform operations on rational expressions <br> Ive rational equations and identify extraneous solutions <br> aph rational functions using critical attributes (asymptotes, shape of curves, intercepts, etc) <br> ind a horizontal asymptote by comparing the degrees of the numerator and denominator |

- Find a vertical asymptote by determining any restricted values for x
- **Honors only**: Find a slant asymptote by using division
- Find $x / y$-intercepts by plugging in $y=0 / x=0$
- Perform graph analysis of rational functions
- Identify and classify types of asymptotes
- State the domain, range, end behavior using proper notation
- Evaluate limits to determine end behaviors
- Describe functions as continuous or discontinuous on a given interval and identify any point(s) of discontinuity
- Compare $f(x)$ to transformational forms: $f(x)+k, k f(x), f(k x)$, and $f(x+k)$ for rational functions
- **Honors only**: Use division to convert from $p(x) / q(x)$ into transformal form
- **Honors only**: Write the equation of a rational function given attributes
- Model real world problems that use rational equations
- Identify the constant of variation for direct, inverse, and joint variation problems
- **Honors only**: Set-up and solve distance, work, and/or mixture problems


## Resources

Core Text: Algebra 2: Big Ideas Learning (2022 Edition by: Ron Larson, Laurie Boswell)
Suggested Resources: Desmos, Delta Math, Kuta Software

## UNIT 8: Statistics

## Summary and Rationale

This unit focuses on data sampling and examinations of the normal curve and the empirical rule of data distribution. Sample spaces and populations will be discussed along with bias and deductive and inductive reasoning. Measures of central tendency, histograms and other graphic display methods, and basic probability relationships will also be explored. Modeling and problem solving will be embedded throughout the unit to ensure that students can transfer their algebraic understandings to real-life scenarios.

## Recommended Pacing

## Academic: $5 \quad$ Honors: 5

## State Standards

| Standard S-CPA Conditional Probability and the Rules of Probability |  |
| :--- | :--- |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 1 | Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of <br> the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). |
| 5 | Recognize and explain the concepts of conditional probability and independence in everyday language and <br> everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the <br> chance of being a smoker if you have lung cancer. |
| Standard S-IC Making Inferences and Justifying Conclusions |  |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 1 | Understand statistics as a process for making inferences about population parameters based on a random <br> sample from that population. |
| 2 | Decide if a specified model is consistent with results from a given data-generating process, e.g., using <br> simulation. |
| 3 | Recognize the purposes of and differences among sample surveys, experiments, and observational studies; <br> explain how randomization relates to each. |
| Standard S-ID Interpreting Categorical and Quantitative Data |  |
| CPI \# | Cumulative Progress Indicator (CPI) |
| 2 | Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and <br> spread (interquartile range, standard deviation) of two or more different data sets. |
| 3 | Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible <br> effects of extreme data points (outliers). |

## Instructional Focus

## Unit Enduring Understandings

- Sample populations and survey questions must be carefully constructed.
- Simple statistical calculations can be used to evaluate reports based on data.
- A normal distribution has the majority of its data centered at the mean, with the standard deviation being a measure of its spread


## Unit Essential Questions

- What are some considerations when undertaking a statistical study?
- How can you use a sample survey to infer a conclusion about a population?
- How can an unbiased sample and an unbiased survey be constructed?
- What are the key characteristics of a normal distribution?


## Objectives

## Students will know:

- Normal distribution and the empirical rule (68/95/99.7)
- $95 \%$ of the data in a normal distribution curve will fall within 2 standard deviations of the mean
- Histogram analysis and recognizing skewed distributions
- Outliers will skew data
- A data display will give you the shape of the distribution
- The mean and the median show you the center of the distribution
- The standard deviation and the interquartile range gives you the spread of the distribution
- Experimental vs. theoretical probability (time permitting)
- Population vs. sample (time permitting)
- Different types of sampling methods and bias in sampling (time permitting)

Vocabulary: mean, standard deviation, probability, compound event, complement, independent and dependent events, empirical rule, population, parameter, simple random sampling, self-selected sampling, convenience sampling, systematic sampling, stratified sampling, cluster sampling, uniform distribution, normal distribution, skewed distribution

## Students will be able to:

- Use measures of central tendency and measures of dispersion to describe data sets
- Calculate probabilities using normal distributions
- Draw a normal distribution curve and commit to memory the empirical rule
- Interpret normal distributions
- Calculate probabilities given 2-way frequency tables
- Interpret box-and-whisker plots and histograms
- Use the fundamental counting principle to count the number of ways an event can happen
- Find theoretical and experimental probabilities.
- Find the probability of independent and dependent events
- Differentiate between the different sampling types


## Resources

Core Text: Algebra 2: Big Ideas Learning (2022 Edition by: Ron Larson, Laurie Boswell)

Suggested Resources: Desmos, Delta Math, Kuta Software

